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Theoretical analysis of generation of acoustic waves in water by axially asymmetric laser beams

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Abstract : The paper presents an analysis of ultrasonic wave generation in water by intensity modulated axially asymmetric TEM_{01} doughnut shaped laser beam, elliptical Gaussian laser beam and an array of *n* identical elliptical Gaussian laser beams It is seen that the generated acoustic radiation is highly directional, both in polar and azimuthal directions. An increase in the asymmetry of the transverse intensity distribution of the laser beam enhances the directivity of the acoustic radiation considerably An obvious conclusion is that axially asymmetric laser beams, particularly an array of multiple asymmetric laser beams are better suited to signal transmission in reasonably well defined solid angle

Keywords : Laser generation of sound, thermoacoustic source (TS), directional acoustic waves, far-field directivity pattern

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1. Introduction

The generation of sound in water by an intensity modulated laser beam has been a subject of considerable interest over the last three decades, because it opens up a new field of air to sea communications in addition to optoacoustic spectroscopy [1], optoacoustic nondestructive testing [2], thermoacoustic medical diagnostics [3], scanning laser acoustic microscopy [4] etc.

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There are several mechanisms through which optical energy is converted to acoustic energy. The thermal mechanism however, offers a great advantage over others because it is easily controllable and hence used in most potential applications of laser acoustics. The physics of the process can be easily understood; when an intensity modulated laser beam passes through water it gives rise to periodic (because of intensity modulation of the beam) heating of the medium, producing a sequence of condensations and rarefactions, propagating away from the exposed region of the medium. Thus, it produces an exponentially varying sound source along its direction of propagation. The frequency of the acoustic wave thus generated, is the same as the modulation frequency of the laser beam. Such acoustic sources are known as thermoacoustic sources (TS).

Characteristics of TS generated by the absorption of intensity modulated CW laser beams by a water body have been studied by Westervelt and Larson [5], Lyamshev and Sedov [6] and Berthelot and Busch-Vishniac [7]. Experimental verification of thermoacoustic radiation have been carried out by various workers [8,9]. Excitation of acoustic pulses by pulsed laser and moving pulsed laser sources have also been investigated [10-15]. Readers are referred to Lyamshev and Sedov [6] for a review of some of the earlier work in this area.

A common feature of these investigations is the study of the effect of Gaussian intensity distribution in the laser beam on the directional characteristics of the emitted acoustic field. Though such a laser beam produces a highly directional acoustic array, its directivity pattern is symmetric around the acoustic array. This symmetry of directivity pattern is a result of the symmetry of the intensity distribution of the laser beam around its axis. Naturally, laser beams with axially asymmetric intensity distribution will produce acoustic fields which will depend both on polar and azimuthal angles. Since the energy is confined within a small solid angle, the range is enhanced in certain directions and as a result such sources would be favourable in comparison to symmetric ones for the purpose of communication, in certain preferred directions.

It is well known that conversion efficiency of the thermoacoustic process is extremely poor. Of course one can increase the intensity of the generated sound by increasing the laser power However, it should be kept in mind that above a certain threshold power density of laser beams, sound generation is not governed by thermoelastic mechanism. In this regime, the sound generation turns out in general to be a nonlinear process manifested by surface evaporation, explosive boiling and optical breakdown. Of course the nonlinear effects associated with high power density lasers can be avoided by dividing the original beam in parts each of which remains within the thermoelastic regime of sound generation. The resulting multiple source configuration could be so arranged as to produce a highly directional acoustic wave. Berthelot [16] has proposed a TS configuration using multiple laser beams. Along this line we propose a TS configuration using n elliptical Gaussian laser beams. The advantage of this configuration over that of n circular Gaussian laser beams is the realisation of improved directivity, a consequence of axially asymmetric intensity distribution of each source.

In this paper, we have investigated far field directivity pattern of TS produced by intensity modulated laser beams. Three different laser configurations namely (i) TEM_{01} doughnut shaped laser beam, (ii) an elliptical Gaussian laser beam, (iii) n equally spaced identical elliptical Gaussian laser beams have been considered. In Section 2 expressions for acoustic pressure generated by a TEM_{01} doughnut shaped laser beam and an elliptical Gaussian laser beam are given. Analysis of acoustic pressure generated by an array of n elliptical Gaussian laser beams is presented in Section 3. The last section outlines the main conclusions.

2. Generation by TEM_{01} doughnut shaped laser beam

Consider an intensity modulated laser beam propagating in the positive z-direction and incident normally on the free air-water interface at $z = 0$. We assume that the laser beam is of moderate power so that change of aggregate state of the medium does not take place in the laser absorption zone and hence, the sound generation mechanism is purely thermal. Absorption of laser produces a thermoacoustic array as shown in Figure 1. The sound pressure field p satisfies the inhomogeneous Helmholtz equation [5]

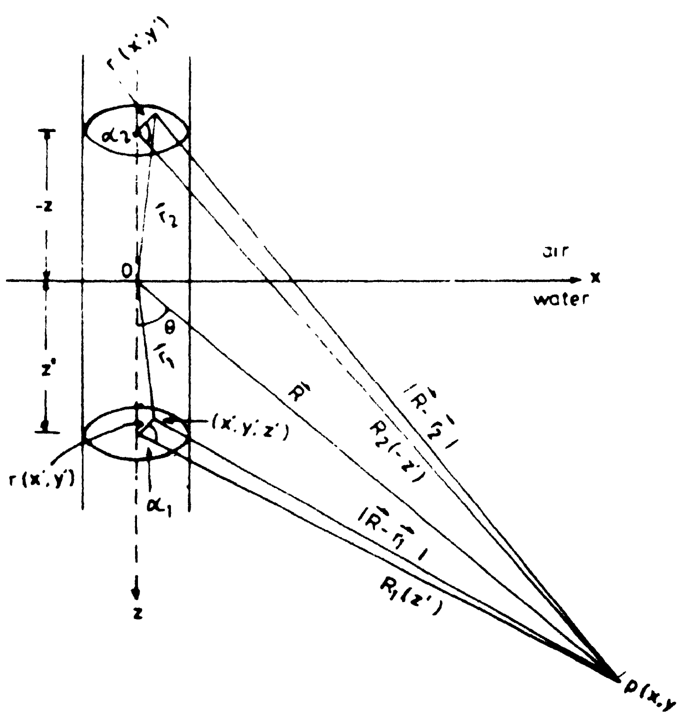

Figure 1. Schematics of thermoacoustic array.

$$\nabla^2 p - \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} \qquad \frac{\beta}{c_p}\frac{\partial q}{\partial t} \tag{1}$$

where c is the speed of sound in water, β is coefficient of thermal expansion, c_p the specific heat at constant pressure and q the amount of heat added in water per unit volume per unit time. The pressure p must satisfy the boundary condition

$$p = 0 \text{ at } z = 0. \tag{2}$$

Heat deposition q in the array region may be written as

$$q\ (x, y, z, t) = A\,\alpha\, I\ (x,y)\ \exp(-\alpha z)(1 + m\cos \omega t), \tag{3}$$

where A is the transmittivity of laser light in water, α the attenuation coefficient, ω the modulation frequency and m the modulation index ($0 < m < 1$). For TEM$_{01}$ doughnut laser beam I may be written as

$$I = I_0 \frac{x^2}{x_0^2} \exp\left[-\left(\frac{x^2+y^2}{x_0^2}\right)\right], \tag{4}$$

where I_0 and x_0 are constants. The beam has an irradiance minimum at $x = 0$, $y = 0$ and irradiance maximum of the beam is at the points $(x_0, 0)$ and $(-x_0, 0)$. In view of eq. (3), eq. (1) may be rewritten as

$$\nabla^2 p + k^2 p = \frac{i\omega m A\alpha\beta}{c_p} I\ (x,y)\ e^{-\alpha z}, \tag{5}$$

where $k = \omega / c$. The time factor $\exp(-i\omega t)$ is implicit in eq. (5) and only the real part of p is the physical solution.

The solution of eq. (5) may be written as

$$p = -\frac{i\omega m A\alpha\beta}{c_p} \int G\,(R/R')\, I\,(x',y')\ \exp\ (-\alpha z')\ dv', \tag{6}$$

where G $(R\ /\ R')$ is the Green's function of the wave equation (eq. (5)). The Green's function which satisfies the required boundary condition at $z = 0$ may be written as

$$G = \frac{1}{4\pi}\left(\frac{e^{ik|R-r_1|}}{|R-r_1|} - \frac{e^{ik|R-r_2|}}{|R-r_2|}\right). \tag{7}$$

Since we are interested in the far field acoustic pattern, the following approximations may be appropriate (see Figure 1)

$$\begin{aligned} |R-r_1| &\simeq R_1\ (z') - r\,(x',y')\ \cos\alpha_1, \\ |R-r_2| &\simeq R_2\ (-z') - r\ (x',y')\ \cos\alpha_2, \end{aligned} \tag{8}$$

where

$$R_1\,(z') = \sqrt{(x^2+y^2+(z-z')^2)}, \quad R_2(-z') = \sqrt{(x^2+y^2+(z+z')^2)}$$

$$r\,(x', y') = \sqrt{(x'^2 + y'^2)},$$

α_1 and α_2 are respectively the angles which $R_1(z')$ and $R_2(-z')$ make with $r(x',y')$. With these substitutions eq. (6) yields

$$p = -\frac{Am\omega\alpha\beta}{2\pi c_p}\frac{e^{ikR}}{R}F(\theta,\phi), \tag{9}$$

where the directivity pattern $F(\theta,\phi)$ is given by

$$F(\theta,\phi) = \left(\frac{\pi x_0^2 I_0}{2}\right)\frac{k\cos\theta}{\alpha^2 + k^2\cos^2\theta}\left(1 - \frac{k^2x_0^2}{2}\sin^2\theta\cos^2\phi\right)$$

$$\times\exp\left[-\frac{k^2x_0^2}{4}\sin^2\theta\right], \tag{10}$$

ϕ is the angle between xz plane and the plane containing the field point and the z axis. The directivity pattern normalised with the directivity pattern of the acoustic field produced by a Gaussian laser beam of equal power may be reduced to

$$F_N(\theta,\phi) = \left[1 - \frac{k^2a_0^2}{2}(x_0/a_0)^2\sin^2\theta\cos^2\phi\right]\exp\left[\frac{k^2a_0^2}{4}\sin^2\theta\right.$$

$$\left.\times\left(1 - \frac{x_0^2}{a_0^2}\right)\right], \tag{11}$$

where a_0 is the radius of the Gaussian beam. In case $x_0/a_0 < 1$ and

$$\left|1 - \frac{k^2a_0^2}{2}(x_0/a_0)^2\sin^2\theta\cos^2\phi\right| > 1 \text{ we must have } |F_N| > 1.$$

For elliptical Gaussian laser beams the intensity distribution $I(x, y)$ may be expressed as

$$I = I_{e0}\exp\left[-\left(\frac{x^2}{a^2} + \frac{y^2}{b^2}\right)\right], \tag{12}$$

where I_{e0} is the intensity on the axis and a and b ($a > b$) are two constants. The acoustic pressure generated by elliptical Gaussian laser beam may be obtained from eq. (6) by using eqs. (7), (8) and (12). Thus

$$p = -\frac{Am\omega\alpha\beta}{2\pi c_p}\frac{e^{ikR}}{R}F(\theta,\phi), \tag{13}$$

where

$$F(\theta,\phi) = \frac{\pi I_{e0}abk\cos\theta}{\alpha^2 + k^2\cos^2\theta}\exp\left[-\frac{k^2a^2\sin^2\theta}{4}\left(\cos^2\phi + \frac{b^2}{a^2}\sin^2\phi\right)\right]. \tag{14}$$

The directivity pattern normalized with the directivity pattern of the acoustic field produced by a Gaussian laser beam of equal power may be written as

$$F_N(\theta,\phi) = \exp\left[k^2a^2\sin^2\theta \left(\frac{b}{a} - \cos^2\phi - \frac{\;}{a^2}\sin^2\phi \right) \right] \tag{15}$$

In order to have a feeling about the directivity pattern of TS, we have carried out numerical calculations of eqs. (11) and (15). Figure 2 displays normalized azimuthal directivity pattern

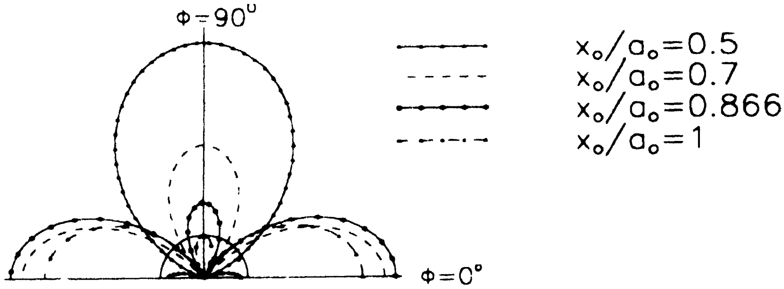

Figure 2. Azimuthal directivity pattern (TS formed by TEM_{01} laser beam) as a function of ϕ for f = 75 kHz, θ = 75° and a_0 = 1 cm

of the acoustic wave generated by a TEM_{01} doughnut laser beam modulated at a frequency $f = 75$ kHz ($f = \omega/2\pi$) and the field point located at a polar angle $\theta = 75^0$. The solid semicircle corresponds to the field produced by a circular Gaussian laser beam of equal power. It is evident that azimuthal directivity is highly sensitive to x_0/a_0 values. For $x_0/a_0 = 1$, acoustic field along the entire azimuthal direction is either smaller than or equal to the field produced by a Gaussian laser beam of equal power. However, as x_0/a_0 decreases from 1, the acoustic field produced by a TEM_{01} laser beam in certain azimuthal direction is enhanced and in certain other azimuthal directions it is diminished in comparison to the Gaussian laser beam. For a small x_0/a_0 ratio, for example for $x_0/a_0 = 0.5$, acoustic radiation is few times greater than that due to a Gaussian laser beam in the azimuthal region $\pi/4 < \phi < 3\pi/4$ and $5\pi/4 < \phi < 7\pi/4$.

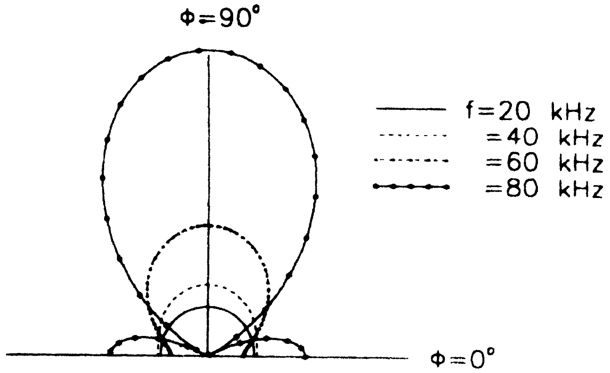

Figure 3. Azimuthal directivity pattern (TS formed by TEM_{01} laser beam) as a function of f for θ = 75°, x_0/a_0 = 0.5 and a_0 = 1 cm

Figure 3 shows variation of normalized directivity pattern with modulation frequency f for $\theta = 75°$ and $x_0/a_0 = 0.5$. It is clear that an increase in modulation frequency increases directivity of acoustic radiation.

Figure 4 shows normalized azimuthal directivity pattern of acoustic wave generated by an elliptical Gaussian laser beam modulated at a frequency f = 60 kHz and the field point is

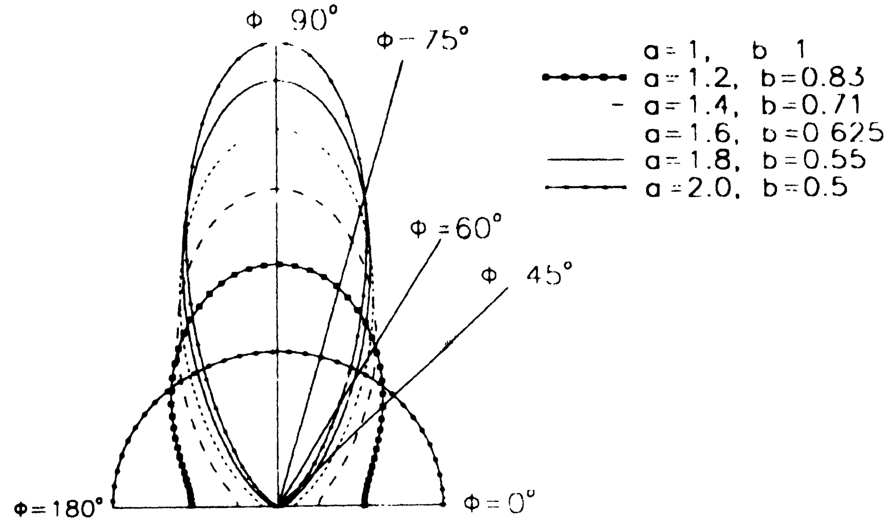

Figure 4. Azimuthal directivity pattern (TS formed by elliptical Gaussian laser beam) as a function of ϕ for f = 60 kHz, θ = 75°.

located at a polar angle 75°. It is clear that azimuthal directivity increases as the asymmetry in the intensity distribution increases (*i.e*, *b*/*a* ratio decreases). For small *b*/*a* ratio acoustic radiation is confined within a small angular region. Figure 5 shows variation of normalized

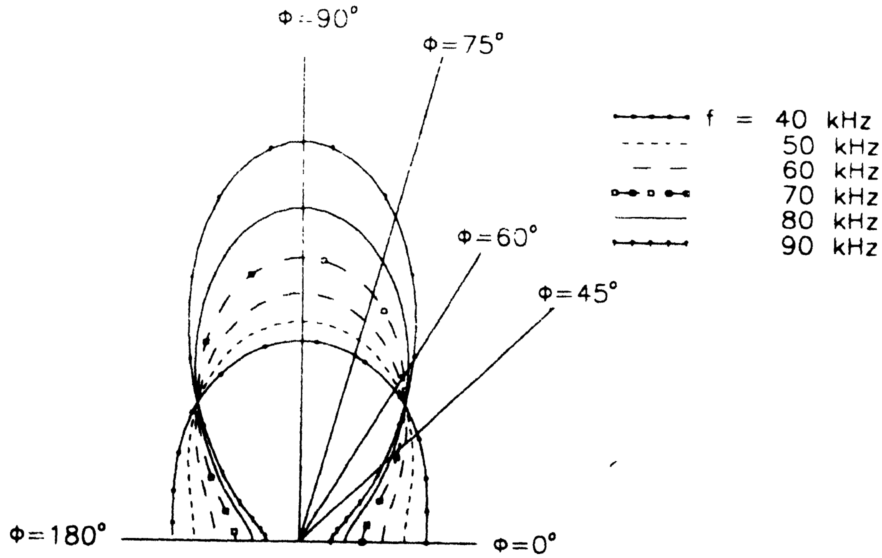

Figure 5. Azimuthal directivity pattern (TS formed by elliptical Gaussian laser beam) as a function of f for θ = 45°, b/a = 0.5

directivity pattern with modulation frequency f for $\theta = 45^0$ and $b/a = 0.5$. It is clear that an increase in f increases directivity of generated acoustic radiation.

3. Generation by n elliptical Gaussian laser beams

Let us now consider the case of n identical elliptical Gaussian laser beams of equal power each separated by a distance d from its neighbour. Let the beams be normally incident on water surface to form a linear array as shown in Figure 6. The acoustic field at a large

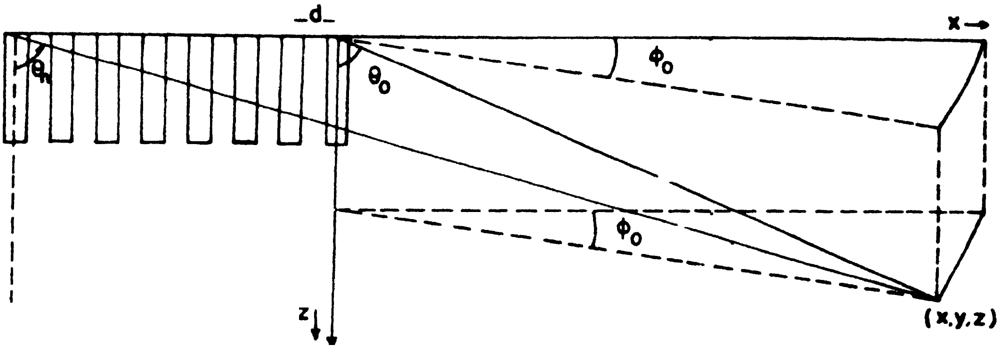

Figure 6. Schematic of n acoustic array

distance produced by such an array of line sources can be obtained by adding the contribution of each source. For each individual source one can use the field expression as given by eq. (13). Therefore, farfield acoustic pressure may be written as

$$p(x,y,z,\omega) = -\frac{Am\omega\alpha\beta}{2\pi c_p}\left(\pi abI_{e0}k\right)\sum_{n=1}^{n}\frac{e^{ikR_n}}{R_n}\frac{k\cos\theta_n}{\alpha^2+k^2\cos^2\theta_n}$$

$$\times\ \exp\left[-\frac{k^2a^2\sin^2\theta_n}{4}\left(\cos^2\phi_n+\frac{b^2}{a^2}\sin^2\phi_n\right)\right], \tag{16}$$

where θ_n is the angle between z axis and R_n, and ϕ_n is the angle between xz plane and the plane containing z axis and R_n. When $nd << R_n$, using the approximations $\cos\phi_n \simeq \cos\phi_0$, $\cos\theta_n \simeq \cos\theta_0$, $\sin\phi_n \simeq \sin\phi_0$, $\sin\theta_n \simeq \sin\theta_0$ and $R_n \simeq R_1 + (n-1)\,d\sin\theta_0\cos\phi_0$, eq. (16) may be rewritten as

$$\frac{|p(R_1)|}{Am\beta I_{e0}ab/2c_p} = \frac{\omega k\alpha\cos\theta_0}{\alpha^2+k^2\cos^2\theta_0}\exp\left[-\frac{k^2a^2\sin^2\theta_0}{4}\right.$$

$$\left.\left(\cos^2\phi_0+\frac{b^2}{a^2}\sin^2\phi_0\right)\right]\frac{\sin(n\Phi/2)}{\sin(\Phi/2)} \tag{17}$$

where $\Phi = kd\sin\theta_0\cos\phi_0$. We have displayed (Figure 7) acoustic radiation as a function of azimuthal angle Φ_0 for $f = 60$ kHz, $\theta = 45^0$, $n = 10$, $d = 1$ cm and $\alpha = 0.137$/cm (corresponds to Nd-glass laser). Acoustic field is confined within a very small azimuthal angle and consequently the linear array behaves as a broad side array and field amplitude increases with the increase in the asymmetry. Figure 8 depicts acoustic radiation as a function

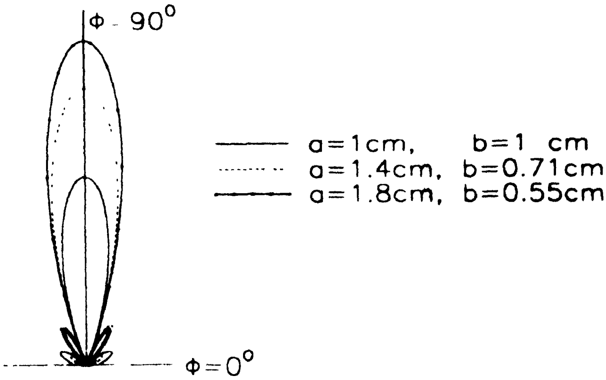

Figure 7. Azimuthal directivity pattern of n array as a function of ϕ for f = 60 kHz, θ = 45°, n = 10, d = 1 cm

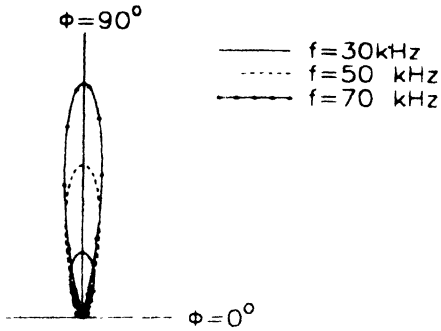

Figure 8. Azimuthal directivity pattern of n array as a function of f for θ = 60°, a = 1 8 cm, b = 0 55 cm, n = 10 cm, d = 1 cm

of f for a = 1.8, b = 0.55 cm, $\theta_0 = 60^0$, n = 10, d = 1 cm and α = 0.137/cm . Qualitative behaviour of field pattern with ϕ is similar as in the previous case.

4. Conclusion

The thermoacoustic mechanism for sound generation by absorption of axially asymmetric laser radiation has been studied for various laser configurations. It is seen that it is possible to generate well collimated sound beams which are collimated not only in the polar direction but in the azimuthal direction as well. Directivity of generated sound is highly sensitive to the asymmetry of intensity distribution of laser beams. Directivity can be increased significantly by using multiple laser beams instead of a single beam. As a consequence of this fact, the range in the direction of confinement is enhanced considerably. Hence, such sources should be preferred in comparison to symmetric sources for more effective communication.

Acknowledgment

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